Numerical Inversion of a 2-Scale Magneto-Elastic Behaviour Model

Laurent Bernard[1,3], Benjamin Joseph Mailhé[1], Nelson Sadowski[1] and Laurent Daniel[2]

[1]GRUCAD/EEL/CTC, Universidade Federal de Santa Catarina, Florianópolis 88040-900, Brazil
[2]GeePs | Group of Electrical Engineering-Paris,
UMR CNRS 8507, CentraleSupélec, Univ. Paris-Sud, Université Paris-Saclay, Sorbonne Universités, UPMC Univ Paris 06,
3 & 11 rue Joliot-Curie, Plateau de Moulon 91192 Gif-sur-Yvette CEDEX, France
[3]Corresponding Author: laurent.bernard@ufsc.br

Magnetic field and mechanical stress are the usual input variables of magneto-mechanical behaviour models. From a 2-scale approach, we obtain a differential model by expressing the derivatives of the magnetization and the magnetostriction strain with respect to the magnetic field and the mechanical stress. The original 2-scale model is then numerically inverted by Newton-Raphson method in order to accept the magnetic flux density and the total strain as input variables.

***Index Terms*—Magneto-Elastic Behaviour, Multi-Scale Modelling, Numerical Inversion**

I. Introduction

MULTI-SCALE approaches allow to derive predictive models to describe the complex magneto-elastic behaviour of ferromagnetic materials. The introduction of such couplings coming from material behaviour in system modelling has taken a growing importance as multiphysical solicitations appear more strongly in high performance design. Indeed, in many systems mechanical and magnetic phenomena must be treated together in a single problem. A simplified multi-scale approach using two scales (magnetic domain and single crystal) was shown to be suitable for the simulation of systems in association with a finite element solver for example [1], [2]. This Simplified Multi-Scale Model (SMSM) is derived from a more complete multi-scale approach [3] in order to reduce computational cost. The SMSM naturally takes the magnetic field ($\vec{H}$) and the mechanical stress ($\boldsymbol{\sigma}$) as input variables. However, in the simulation of systems by finite element method, the magneto-mechanical problem is generally expressed in terms of magnetic flux density ($\vec{B}$) and total mechanical strain ($\boldsymbol{\varepsilon}$) through the usual displacement and magnetic vector potential formulations. To solve the fully coupled non-linear problem the model of material behaviour must accept $\vec{B}$ and $\boldsymbol{\varepsilon}$ as input variables. We propose here a numerical inversion of the SMSM: first the model is differentiated with respect to the natural input variables and then the Newton-Raphson method is applied.

II. Differential SMSM

In the SMSM the material is modeled as a fictitious single crystal made of a collection of magnetic domains randomly oriented [4]. At the scale of a magnetic domain, the local magnetization ($\vec{M}_\alpha$) and magnetostriction strain ($\boldsymbol{\varepsilon}^\mu_\alpha$) depend only on the orientation of the magnetization ($\vec{\alpha}$) in the domain and on the saturation magnetization (M_s) and maximum magnetostrictive strain (λ_s). The local potential energy of the domains oriented along $\vec{\alpha}$ is the sum of a magnetic, an elastic and anistropy contributions:

$$W_\alpha = W_\alpha^{mag} + W_\alpha^{el} + W_\alpha^{an} \tag{1}$$

with

$$W_\alpha^{mag} = -\mu_0 \vec{H}.\vec{M}_\alpha \tag{2}$$

$$W_\alpha^{el} = -\boldsymbol{\sigma} : \boldsymbol{\varepsilon}^\mu_\alpha \tag{3}$$

$$W_\alpha^{an} = J(\vec{\alpha}.\vec{\beta})^2 \tag{4}$$

and where μ_0 is the vacuum permeability, $\vec{H}$ is the applied magnetic field, $\boldsymbol{\sigma}$ is the applied stress tensor, $\vec{\beta}$ is the anistropy direction and J is the anisotropy constant. The volume fraction of each domain family (f_α) is calculated assuming a Boltzmann-type distribution [5] with respect to the domain energies:

$$f_\alpha = \frac{\exp\left(-A_s W_\alpha\right)}{\displaystyle\int \exp\left(-A_s . W_\alpha\right) d\alpha} \tag{5}$$

Finally, the macroscopic anhysteretic magnetization and magnetostriction strains are obtained thanks to an averaging operation over all possible directions

$$\vec{M} \; = < \vec{M}_\alpha > = \int f_\alpha \, \vec{M}_\alpha \, d\alpha \tag{6}$$

$$\boldsymbol{\varepsilon}^\mu \; = < \boldsymbol{\varepsilon}^\mu_\alpha > = \int f_\alpha \, \boldsymbol{\varepsilon}^\mu_\alpha \, d\alpha \tag{7}$$

To obtain the differential model the derivatives of $\vec{M}$ and $\boldsymbol{\varepsilon}$ with respect to the input variables $\vec{H}$ and $\boldsymbol{\sigma}$ are expressed analytically. For example the differential susceptibility tensor, which is the derivative of $\vec{M}$ with respect to $\vec{H}$ at constant stress $\boldsymbol{\sigma}$ is obtained by differentiating (6). Because $\vec{M}_\alpha$ and $\vec{\alpha}$ do not depend on the magnetic field, we have:

$$\partial_{\vec{H}} \vec{M} = \int \partial_{\vec{H}} f_\alpha \otimes \vec{M}_\alpha \, d\alpha \tag{8}$$

It can be shown that the partial derivative of f_α with respect to $\vec{H}$ is:

$$\partial_{\vec{H}} f_\alpha = A_s \left(-f_\alpha \partial_{\vec{H}} W_\alpha + f_\alpha \int f_\alpha \partial_{\vec{H}} W_\alpha d\alpha \right) \tag{9}$$

From equation (1) to (4), we also have:

$$\partial_{\vec{H}} W_\alpha = -\mu_0 \vec{M}_\alpha \tag{10}$$

Finally, the analytical expression for the differential susceptibility is obtained as:

$$\partial_{\vec{H}} \vec{M} = \mu_0 A_s \left(\int f_\alpha \vec{M}_\alpha \otimes \vec{M}_\alpha \, d\alpha - \vec{M} \otimes \vec{M} \right) \tag{11}$$

It can be noticed from equation (11) that the differential susceptibility tensor is proportional to the difference between the tensor product of the macroscopic magnetization by itself and the volume fraction weighted average of the tensor product of the local magnetization by itself. The other components of the differential model can be obtained in the same way:

$$\partial_{\boldsymbol{\sigma}} \boldsymbol{\varepsilon}^\mu = A_s \left(\int f_\alpha \boldsymbol{\varepsilon}^\mu_\alpha \otimes \boldsymbol{\varepsilon}^\mu_\alpha \, d\alpha - \boldsymbol{\varepsilon}^\mu \otimes \boldsymbol{\varepsilon}^\mu \right) \tag{12}$$

$$\partial_{\boldsymbol{\sigma}} \vec{M} = A_s \left(\int f_\alpha \boldsymbol{\varepsilon}^\mu_\alpha \otimes \vec{M}_\alpha \, d\alpha - \boldsymbol{\varepsilon}^\mu \otimes \vec{M} \right) \tag{13}$$

$$\partial_{\vec{H}} \boldsymbol{\varepsilon}^\mu = A_s \mu_0 \left(\int f_\alpha \vec{M}_\alpha \otimes \boldsymbol{\varepsilon}^\mu_\alpha \, d\alpha - \vec{M} \otimes \boldsymbol{\varepsilon}^\mu \right) \tag{14}$$

The last two tensors are the same except for a factor μ_0 and a transposition, i.e., $\mu_0 \partial_{\boldsymbol{\sigma}} \vec{M}_{ijk} = \partial_{\vec{H}} {\boldsymbol{\varepsilon}^\mu}_{jki}$. The set of equations (11) to (14) constitutes the output of differential SMSM.

III. Inverse model

From the differential model, the inverse SMSM model can be obtained numerically. Using Voigt's notation for the stress and strain tensors, the differential model can be written in matrix form as:

$$\begin{bmatrix} \mu_0 d\vec{M} \\ d\boldsymbol{\varepsilon}^\mu \end{bmatrix} = \begin{bmatrix} \mu_0 \partial_{\vec{H}} \vec{M} & \mu_0 \partial_{\boldsymbol{\sigma}} \vec{M} \\ \partial_{\vec{H}} \boldsymbol{\varepsilon}^\mu & \partial_{\boldsymbol{\sigma}} \boldsymbol{\varepsilon}^\mu \end{bmatrix} \begin{bmatrix} d\vec{H} \\ d\boldsymbol{\sigma} \end{bmatrix} = \boldsymbol{F} \begin{bmatrix} d\vec{H} \\ d\boldsymbol{\sigma} \end{bmatrix} \tag{15}$$

To inverse the model, we need to find $\left(\vec{H}, \boldsymbol{\sigma} \right)$ such that:

$$\vec{B} = \mu_0 \left(\vec{H} + \vec{M} \right) \tag{16}$$

and

$$\boldsymbol{\varepsilon} = \boldsymbol{S}\boldsymbol{\sigma} + \boldsymbol{\varepsilon}^\mu \tag{17}$$

where $\vec{B}$ and $\boldsymbol{\varepsilon}$ are the applied magnetic flux density and total strain. Using Newton-Raphson method, an approximate solution is found by solving iteratively:

$$\begin{bmatrix} \delta\vec{H} \\ \delta\boldsymbol{\sigma} \end{bmatrix} = -\boldsymbol{G}^{-1} \begin{bmatrix} \delta\vec{B} \\ \delta\boldsymbol{\varepsilon} \end{bmatrix} \tag{18}$$

where

$$\boldsymbol{G} = \begin{bmatrix} \mu_0 \boldsymbol{I}_1 & 0 \\ 0 & \boldsymbol{S} \end{bmatrix} + \boldsymbol{F} \tag{19}$$

and $\begin{bmatrix} \delta\vec{B} \\ \delta\boldsymbol{\varepsilon} \end{bmatrix}$ is the residual. For the numerical evaluation, the integral terms appearing in the SMSM and in the differential model are computed as discrete sums over a set of 2562 almost uniformly distributed possible orientations [3]. The iterative Newton-Raphson procedure is stopped when the relative variation of the norm of the residual is less than 10^{-4} or when the number of iterations exceeds 30. The parameters used for the model are given in Table I. As an example, we show the map of the (x, x, x) component of tensor $\partial_{\boldsymbol{\varepsilon}} \vec{H}$ with respect to the $x-$ and $y-$components of the induction flux density obtained from the inverse model and the corresponding number of iterations (Fig. 1). The maps are obtained by nested loops incrementing the component of the magnetic flux density ($y-$component is incremented in the inner loop), starting from $\vec{B} = 0$ and using last solution as initial guess in the iterative process. The maps show the expected behaviour. It can be seen that the number of iterations is generally low (less than 5) but that the algorithm does not converge when the material is highly saturated.

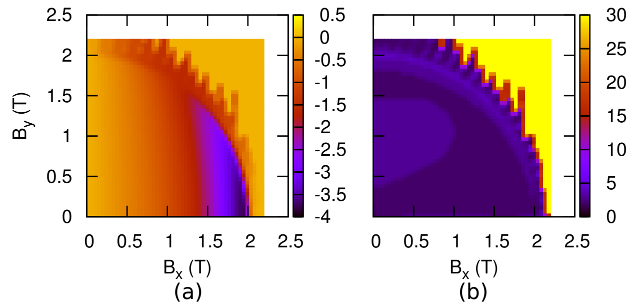

Fig. 1. Map of (a) $\partial_{\boldsymbol{\varepsilon}} \vec{H}_{xxx}$ (10^6 A/m) and (b) number of iterations of the Newton-Raphson algorithm, as a function of components $\vec{B}_x$ and $\vec{B}_y$ of the magnetic flux density

TABLE I
Parameters for the multiscale model.

$A_s (m^3/J)$	$M_s (A/m)$	λ_s	$J (J/m^3)$
10^{-2}	$1.6\,10^6$	10^{-5}	0

IV. Conclusion

The approach proposed here allows the numerical inversion of the 2-scale model of magneto-elastic behaviour. With this inverse model in hand, coupled magneto-elastic simulation of systems using classical formulations can be managed. Convergence of the material model might not be reached in highly saturated regions which would require a special treatment. Next step on the way will be the application to finite element simulations.

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